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TABLES OF TRANSITION PROBABILITIES AND BRANCHING RATIOS FOR ELECTRIC DIPOLE TRANSITIONS BETWEEN ARBITRARY LEVELS OF HYDROGEN-LIKE ATOMS

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All measurement values are expressed in the International System of Units (SI) in accordance with NASA Policy Directive 2220.4, paragraph 4.

TABLES OF TRANSITION PROBABILITIES AND BRANCHING RATIOS FOR ELECTRIC DIPOLE TRANSITIONS BETWEEN ARBITRARY LEVELS OF HYDROGEN-LIKE ATOMS

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ABSTRACT

Branching ratios in hydrogen-like atoms due to electric-dipole transitions are tabulated for the initial principal and angular momentum quantum number $n'\ell'$, and final principal and angular momentum quantum numbers $n\ell$. In Table 1, transition probabilites are given for transitions $n'\ell' \rightarrow n$, where sums have been made with respect to ℓ. In this table, $2 \leq n' \leq 10$, $0 \leq \ell' \leq n'-1$, and $1 \leq n \leq n'-1$. In addition, averages with respect to ℓ' and sums with respect to n, and lifetimes are given.

In Table II, branching ratios are given for transitions $n'\ell' \rightarrow n$, where sums ave been made with respect to ℓ. In these tables, $2 \leq n' \leq 10$, $0 \leq \ell' \leq n'-1$, and $1 \leq n \leq n'-1$. Averages with respect to ℓ' are also given.

In Table III, branching ratios are given for transitions $n'\ell' \rightarrow n\ell$, where $1 \leq n \leq 5$, $0 \leq \ell \leq n-1$, $n < n' \leq 15$, and $0 \leq \ell' \leq n_s$, where n_s is the smaller of the two numbers $n'-1$ and 6. Averages with respect to ℓ' are also given.

I. INTRODUCTION

Excited states of atoms and ions are formed frequently in laboratory and astrophysical plasmas. The important processes leading to the atomic excited states are the radiative and dielectronic recombinations in plasmas, electron capture by ions through the charge exchange collisions with atoms and ions, and collisional excitation of atoms and ions with electrons and heavy charged or neutral particles. The dominant transitions for cascading of the excited states are the electric dipole transitions. It is convenient to have tables of values for the branching ratios for these transitions between the important initial and final levels. This tabulation is being done here for the case of the hydrogen-like atoms.

In Section II calculation of the branching ratios are formulated and in Section III three tables are given.

In Table I transition probabilities for a number of initially excited hydrogen-like atoms whose range of quantum numbers are described in the abstract are given. This table is an extension of a similar table given by Bethe and Salpeter[1], and is given here not for its novelty, but for its differences with similar tabulations. Related quantities, the squares of the radial integrals in the matrix elements of the electric dipole moment for various transitions, have been tabulated extensively by Green, Rush, and Chandler.[2] However, these tabulations are for transitions of the type $n'\ell' \rightarrow n\ell$, and various sums and averages with respect to n, ℓ, and ℓ' are not given. Similarly, Hiskes and Tarter[3] tabulate transition probabilities for transitions of the type $n'\ell' \rightarrow n\ell$ and $n'\ell' \rightarrow n$. They also give lifetimes of various excited states $n'\ell'$ in graphical forms. As a result, some numbers given in Table I are repetitions of the numbers and graphs given by Hiskes and Tarter.

Tables II and III are completely new, and give branching ratios between various transitions and their sums and averages.

The values for the branching ratios for the low-lying levels can easily be obtained using the known values of the atomic electric dipole moments for various transitions. However, when the initial state is a highly excited state, due to numerous ways that the electron can reach lower levels, the calculation becomes cumbersome. Tables II and III alleviate this difficulty by tabulating branching ratios between different excited states. Table II gives branching ratios for transitions of the type $n'\ell' \rightarrow n$, and averages with respect to ℓ'. Table III gives branching ratios for transitions of the type $n'\ell' \rightarrow n\ell$, and averages with respect to ℓ'. The range of the quantum numbers covered are given in the abstract.

Tables presented in this paper are for an isolated hydrogen-like atom free of any perturbations. However, states with high n values are very easily perturbed by small electrostatic fields, which split the n^2-fold hydrogen-like degeneracy into a number of "Stark states" (first order Stark effect). This can drastically change the selection rules and branching ratios. Thus it is possible that the results presented here may correspond to a situation which is not realizable experimentally. In a forthcoming paper, transition probabilities and branching ratios between Stark levels of hydrogen-like atoms are tabluated and presented.

II. FORMULATION

The electric dipole transition probability for the initial and final states i and f is given by the Einstein's A coefficient:[1]

$$A(f, i) = \frac{1}{3}\alpha^3 (R_\infty/\hbar a_0^2)(\Delta E/R_\infty)^3 \tag{1}$$

$$\times |\langle i | \underset{\sim}{r} | f \rangle|^2,$$

where $\langle i | \underset{\sim}{r} | f \rangle$ is the electric dipole moment, $\Delta E/R_\infty$ is the energy difference between the levels in rydberg, and α and a_0 are the fine structure constant and the Bohr radius. For the case of the hydrogen-like atoms (1) reduces to

$$A(n\ell, n'\ell') = \frac{1}{3}\alpha^3 (R_\infty/\hbar a_0^2)\ \mu Z^4 \left(\frac{1}{n^2} - \frac{1}{n'^2}\right)^3$$

$$\times \sum_{mm'} |\langle n\ell m | \underset{\sim}{r} | n'\ell' m' \rangle|^2 \tag{2}$$

where $n'\ell'$ and $n\ell$ represent the principal and angular momentum quantum numbers of the initial and final states, μ is the reduced mass, and Z is the nuclear charge of the atom. m and m′ are the

magnetic quantum numbers of the final and initial states, and $|\langle n\ell\, m\, |\underset{\sim}{r}|\, n'\ell'\, m'\rangle|^2$ is the square of the electric dipole moment for the atomic hydrogen.

The branching ratio due to the direct transition between $n'\ell'$ and $n\ell$, which we call $\beta(n\ell, n'\ell')$, is given by

$$\beta(n\ell, n'\ell') = \frac{A(n\ell, n'\ell')}{\sum_{n''=1}^{n'-1} \sum_{\ell''=0}^{n''-1} A(n''\ell'', n'\ell')} \tag{3}$$

However, many transitions from $n'\ell'$ to $n\ell$ take place by the transient electron passing through the intermediate states $n''\ell''$ before reaching $n\ell$. Let us designate the branching ratio due to all possible transitions by $\beta_T\,(n\ell, n''\ell'')$. Then if the values of $\beta_T\,(n\ell, n''\ell'')$ are known for all values of $n''\ell''$ up to $n'' < n'$-1, the value of $\beta_T\,(n\ell, n'\ell')$ is obviously given by the following formula:

$$\beta_T\,(n\ell, n'\ell') = \beta(n\ell, n'\ell') + \sum_{n''=n+1}^{n''-1} \sum_{\ell''=0}^{n''-1} \beta(n''\ell'', n'\ell')\,\beta_T(n\ell, n''\ell'') \tag{4}$$

Of usefulness are also the branching ratios obtained by averaging $\beta_T\,(n\ell, n'\ell')$ with respect to ℓ' and summing with respect to ℓ:

$$\beta_T\,(n\ell, n') = \frac{1}{n'^2} \sum_{\ell'=0}^{n'-1} (2\ell'+1)\,\beta_T(n\ell, n'\ell'), \tag{5}$$

$$\beta_T\,(n, n') = \sum_{\ell=0}^{n-1} (2\ell+1)\,\beta_T\,(n\ell, n') \tag{6}$$

In practice, for a given $n\ell$ Equation (4) is used to calculate $\beta_T\,(n\ell, n'\ell')$ for $n' = n + 1$, keeping in mind that $\beta_T\,(n\ell, n+1\ell') = \beta(n\ell, n+1\ell')$. Then (4) is used successively to obtain the total branching ratio for any desired $n'\ell'$.

In evaluating β_T $(n\ell, n'\ell')$ the values of the dipole moments between $n'\ell'$ and $n\ell$ are needed. Gordon[4] has given a general formula for the electric dipole moment matrices in terms of a hypergeometric function. An alternative derivation will be given below where these matrices are given in terms of finite sums over algebraic expressions.

For convenience let m and m' represent from now on the absolute values of the magnetic quantum numbers. This could be done since the electric dipole moment is invariant with respect to the change of sign of m and m'. Using the expression for $\langle n\ell m \mid \exp(i\underset{\sim}{q}\cdot\underset{\sim}{r}) \mid n'\ell' m'\rangle$, (Ref. 5), we find with straightforward algebra

$$
\langle n\ell m \mid \hat{q}\cdot\underset{\sim}{r} \mid n'\ell' m'\rangle = \delta(m, m')
$$

$$
\times \left[\frac{\partial}{\partial(iq)} \langle n\ell m \mid e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}} \mid n'\ell' m\rangle\right]_{q\to 0} \tag{7}
$$

$$
= \delta(m, m') \sum_{n_1=0}^{n-m-1} \sum_{n'_1=0}^{n'-m-1} \langle n\ell m \mid n n_1 m\rangle \langle n' n'_1 m \mid n'\ell' m\rangle \langle n n_1 m \mid \hat{q}\cdot\underset{\sim}{r} \mid n' n'_1 m\rangle
$$

where $\underset{\sim}{q}$ is an arbitrary vector, $n' n'_1 m$ and $n\, n_1\, m$ are the parabolic coordinate quantum numbers of the initial and final states, $\langle n\, n_1\, m \mid n\ell m\rangle$ are elements of the transformation matrix between the spherical and parabolic coordinates eigenstates. This matrix is related to the Wigner's 3j symbol. The relation was first discovered by Park,[6] and later rediscovered by Hughes,[7] and Barut and Kleinert.[8] It is given by

$$
\langle n n_1 m \mid n\ell m\rangle = (-)^m (2\ell+1)^{1/2} \begin{pmatrix} \frac{1}{2}(n-1) & \frac{1}{2}(n-1) & \ell \\ \frac{1}{2}(m-n_1+n_2) & \frac{1}{2}(m+n_1-n_2) & -m \end{pmatrix} \tag{8}
$$

Finally,

$$\langle nn_1m|\hat{q}\cdot r|n'n'_1m\rangle = \frac{1}{4}\left[\frac{4nn'}{(n+n')^2}\right]^{m+2}$$

$$\times\left[\frac{(n_1+m)!\,(n_2+m)!\,(n'_1+m)!\,(n'_2+m)!}{n_1!\,n_2!\,n'_1!\,n'_2!}\right]^{1/2}$$

$$\times\sum_{\nu_1=0}^{n_1}\sum_{\nu_2=0}^{n_2}\sum_{\nu'_1=0}^{n'_1}\sum_{\nu'_2=0}^{n'_2}\left(\frac{-2n'}{n+n'}\right)^{\nu_1+\nu_2}\left(\frac{-2n}{n+n'}\right)^{\nu'_1+\nu'_2}\binom{n_1}{\nu_1}\binom{n_2}{\nu_2}\binom{n'_1}{\nu'_1}\binom{n'_2}{\nu'_2}$$

$$\times\frac{(m+\nu_1+\nu'_1)!\,(m+\nu_2+\nu'_2)!}{(m+\nu_1)!\,(m+\nu'_1)!\,(m+\nu_2)!\,(m+\nu'_2)!}\,[\lambda_1(\lambda_1+1)-\lambda_2(\lambda_2+1)], \tag{9}$$

$$n_2 = n-m-1-n_1,\; n'_2 = n'-m-1-n'_1,$$

$$\lambda_1 = m+1+\nu_1+\nu'_1,\; \lambda_2 = m+1+\nu_2+\nu'_2$$

Equations (7) to (9) can be combined with

$$|\langle n\ell m|\underset{\sim}{r}|n'\ell'm'\rangle|^2 = 3|\langle n\ell m|\hat{q}\cdot\underset{\sim}{r}|n'\ell'm'\rangle|^2 \tag{10}$$

to obtain values of the square of the dipole moments necessary for evaluation of $A(n\ell, n'\ell')$. Then, Eqs. (3) – (6) can be used to find values of $\beta(n\ell, n'\ell')$, $\beta_T(n\ell, n'\ell')$, $\beta_T(n\ell, n')$ and $\beta_T(n, n')$.

It should be noted that while $A(n\ell, n'\ell')$ increases linearly with the reduced mass of the hydrogen-like atom under consideration and increases as the 4th power of Z with respect to the nuclear charge Z, the branching ratios are independent of these parameters.

III. RESULTS

Results are given in Figure 1 and Tables I through III. In Figure 1, the branching ratios β_T between the initial states $n' = 10$, $\ell' = 0$-9, and the final states $n < n'$ are given as functions of n. To show the variation of β_T versus n, β_T is shown as a continuous function of n, although it has no meaning for non-integer values of n.

It is interesting to note that, except the case $\ell' = 0$, the larger the angular momentum, the larger the branching ratio. This is consistent with the lifetimes of the excited states for different ℓ' where, except for $\ell' = 0$ case, the states with largest ℓ' have longest lifetimes (Hiskes and Tarter[3] and Table I). Physically what it means is that states with high ℓ' values have fewer channels at their disposal for decay, and live longer. In particular, a state with $\ell' = n'-1$ has a single channel for decay (cf. Eq. (11)), and has the longest lifetime. The branching ratios for all transitions to the ground state, except the states with $\ell' = n'-1$, are less than unity due to the metastable state 2s.

In Tables I-III the initial or the upper states are designated by primed, and the final or the lower states are designated by unprimed letters. In Table I transition probabilities are given for the transitions $n'\ell' \rightarrow n$, where sums have been made with respect to ℓ. The principal quantum number n' ranges from 2 to 10, and ℓ' and n take all their possible values. Averages with respect to ℓ' are designated by "Mean", and sums with respect to n are designated by "Total". Similarly, the lifetime of each excited state $n'\ell'$ or n' is given. The transition probabilities of an hydrogen-like atom with a reduced mass μ and an effective charge Z is obtained by multiplying the numbers given in Table I by a factor μZ^4, in accordance with Eq. (2).

In Table II, the branching ratios for transitions $n'\ell' \rightarrow n$, where sums have been made with respect to ℓ, are given. Similarly, the transitions $n' \rightarrow n$ designated by "Mean", where averages are made with respect to ℓ', are given. Ranges of $n'\ell'$, and n are the same as in Table I.

As is seen in the table, the branching ratios for transitions in which $\ell' = n'-1$ are equal to unity. This is due to the fact that if we assume $n'' = n'-1$, and $n''' = n''-1$, then the state with $\ell' = n'-1$ can only decay into the state n'', $\ell'' = n''-1$, and then to the state n''', $\ell''' = n'''-1$, etc. It follows that

$$\beta_T(n\ell, n'\ell' = n'-1) = \delta(\ell, n-1), \qquad (11)$$

as it should be.

In Tables III A through III O, the branching ratios for transitions $n'\ell' \rightarrow n\ell$, and their averages with respect to ℓ', are given. In these tables $1 \leq n \leq 5$, $0 \leq \ell \leq n-1$, $n \leq n' \leq 15$, and $0 \leq \ell' \leq n_s$, here n_s is the smaller of the two numbers $n'-1$ and 6.

For the final states ℓ' where $6 < \ell' < n'-1$ for which the branching ratios are not listed in the tables a crude estimate for the branching ratios could be obtained by interpolating the values of these ratios between values for $\ell' = 6$ given in the tables and $\ell' = n'-1$ given by (11). Similarly, the branching ratios for $n' > 15$ can be crudely estimated by extrapolating the branching ratios for $n' \leq 15$.

All the excited states of the hydrogen-like atoms decay by the electric dipole transitions except the 2s state. Then the sum of the branching ratios of an initial state to the 1s and 2s states should be equal to unity, as is verified in the tables.

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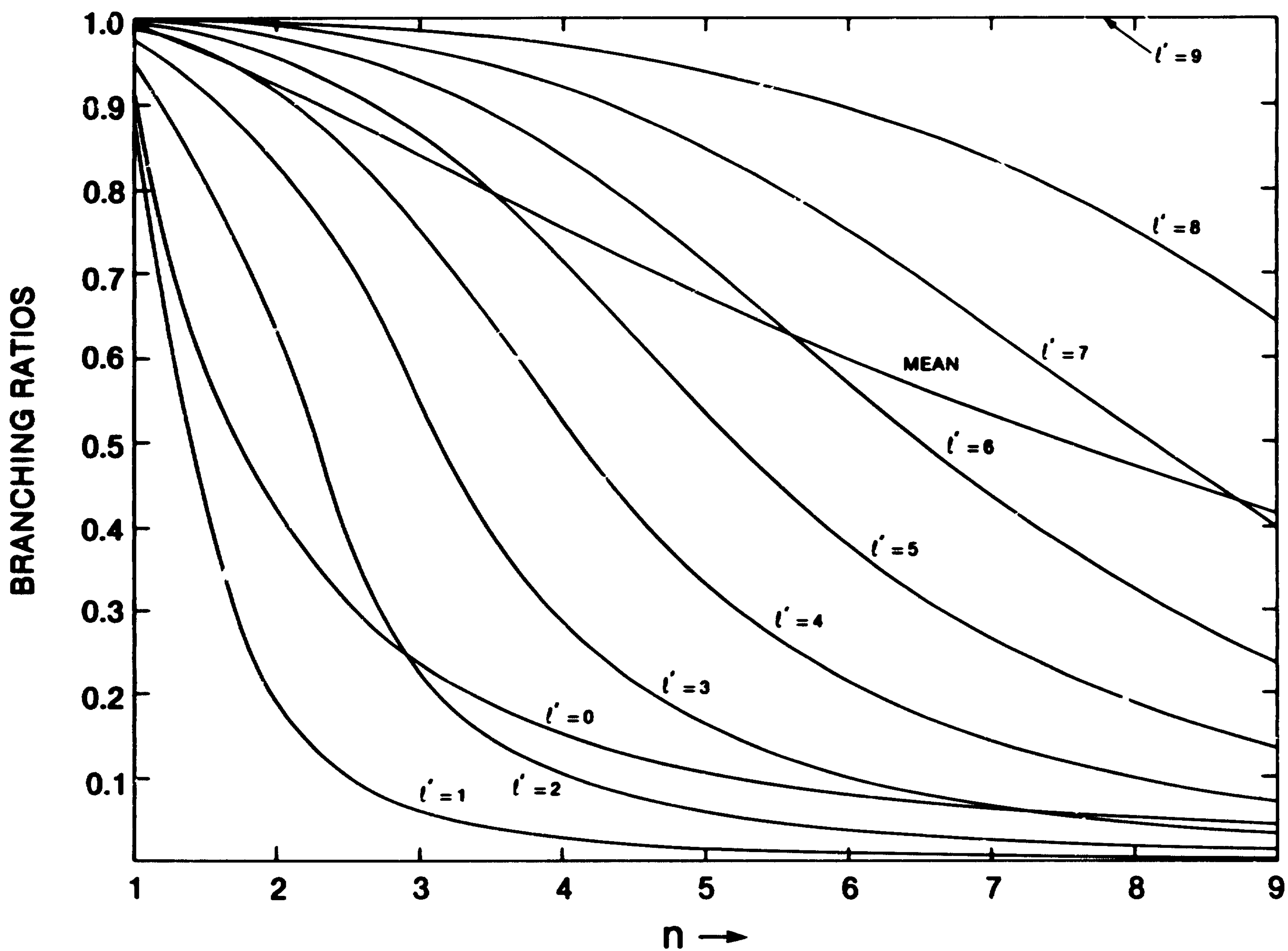

Figure 1. Branching ratios β_T between the initial states n' = 10 and ℓ' = 0-9, and the final states n < n' as functions of n.

Table I
Transition Probabilities in Atomic Hydrogen in sec^{-1}

n'ℓ' \ n	1	2	3	4	5	Total	Lifetime (sec)
20	0.0					0	∞
21	6.27+8*					6.27+8	1.60-9
Mean	4.70+8					4.70+8	2.13-9
30	0.0	6.32+6				6.32+6	1.58-7
31	1.67+8	2.25+7				1.90+8	5.27-9
32	0.0	6.47+7				6.47+7	1.55-8
Mean	5.57+7	4.41+7				1.00+8	1.00-8
40	0.0	2.58+6	1.84+6			4.42+7	2.26-7
41	6.82+7	9.67+6	3.41+6			8.13+7	1.23-8
42	0.0	2.06+7	7.04+6			2.77+7	3.61-8
43	0.0	0.0	1.38+7			1.38+7	7.25-8
Mean	1.28+7	8.42+6	8.99+6			3.02+7	3.31-8
50	0.0	1.29+6	9.05+5	6.45+5		2.84+6	3.52-7
51	3.44+7	4.95+6	1.79+6	9.26+5		4.21+7	2.38-8
52	0.0	9.43+6	3.39+6	1.54+6		1.44+7	6.96-8
53	0.0	0.0	4.54+6	2.59+6		7.13+6	1.40-7
54	0.0	0.0	0.0	4.26+6		4.26+6	2.35-7
Mean	4.13+6	2.53+6	2.20+6	2.70+6		1.16+7	8.65-8
60	0.0	7.35+5	5.07+5	3.58+5	2.68+5	1.87+6	5.35-7
61	1.97+7	2.86+6	1.03+6	5.40+5	3.39+5	2.45+7	4.08-8
62	0.0	5.15+6	1.88+6	8.84+5	4.89+5	8.40+6	1.19-7
63	0.0	0.0	2.15+6	1.29+6	7.35+5	4.17+6	2.40-7
64	0.0	0.0	0.0	1.37+6	1.11+6	2.48+6	4.03-7
65	0.0	0.0	0.0	0.0	1.65+6	1.65+6	6.08-7
Mean	1.65+6	9.74+5	7.79+5	7.72+5	1.03+6	5.21+6	1.93-7

Table I (continued)

n'ℓ' \ n	1	2	3	4	5	6	7	8	Total	Lifetime (sec)
70	0.0	4.59+5	3.13+5	2.17+5	1.62+5	1.27+5			1.28+6	7.83-7
71	1.24+7	1.80+6	6.49+5	3.37+5	2.12+5	1.49+5			1.55+7	6.45-8
72	0.0	3.13+6	1.15+6	5.45+5	3.10+5	1.96+5			5.33+6	1.88-7
73	0.0	0.0	1.21+6	7.34+5	4.38+5	2.71+5			2.65+6	3.77-8
74	0.0	0.0	0.0	6.46+5	5.48+5	3.80+5			1.57+6	6.35-7
75	0.0	0.0	0.0	0.0	5.09+5	5.33+5			1.04+6	9.60-7
76	0.0	0.0	0.0	0.0	0.0	7.41+5			7.41+5	1.35-6
Mean	7.57+5	4.39+5	3.36+5	3.04+5	3.25+5	4.56+5			2.62+6	3.82-7
80	0.0	3.06+5	2.07+5	1.42+5	1.04+5	8.11+4	6.57+4		9.05+5	1.10-6
81	8.26+6	1.20+6	4.34+5	2.24+5	1.40+5	9.82+4	7.41+4		1.04+7	9.59-8
82	0.0	2.05+6	7.56+5	3.59+5	2.04+5	1.32+5	9.14+4		3.60+6	2.78-7
83	0.0	0.0	7.54+5	4.61+5	2.78+5	1.78+5	1.19+5		1.79+6	5.59-7
84	0.0	0.0	0.0	3.64+5	3.14+5	2.28+5	1.58+5		1.06+6	9.41-7
85	0.0	0.0	0.0	0.0	2.34+5	2.58+5	2.10+5		7.02+5	1.42-6
86	0.0	0.0	0.0	0.0	0.0	2.18+5	2.81+5		4.98+5	2.01-6
87	0.0	0.0	0.0	0.0	0.0	0.0	3.73+5		3.73+5	2.68-6
Mean	3.87+5	2.22+5	1.65+5	1.43+5	1.39+5	1.56+5	2.27+5		1.44+6	6.95-7
90	0.0	2.14+5	1.44+5	9.80+4	7.10+4	5.45+4	4.41+4	3.68+4	6.62+5	1.51-6
91	5.79+6	8.44+5	3.04+5	1.56+5	9.67+4	6.73+4	5.08+4	4.03+4	7.35+6	1.36-7
92	0.0	1.42+6	5.24+5	2.49+5	1.42+5	9.10+4	6.41+4	4.76+4	2.54+6	3.94-7
93	0.0	0.0	5.06+5	3.10+5	1.87+5	1.21+5	8.28+4	5.89+4	1.27+6	7.90-7
94	0.0	0.0	0.0	2.29+5	1.98+5	1.45+5	1.05+5	7.48+4	7.52+5	1.33-6
95	0.0	0.0	0.0	0.0	1.30+5	1.46+5	1.25+5	9.60+4	4.97+5	2.01-6
96	0.0	0.0	0.0	0.0	0.0	9.68+4	1.32+5	1.24+5	3.52+5	2.84-6
97	0.0	0.0	0.0	0.0	0.0	0.0	1.04+5	1.59+5	2.63+5	3.81-6
98	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.04+5	2.04+5	4.91-6
Mean	2.14+5	1.22+5	8.91+4	7.46+4	6.91+4	7.07+4	8.24+4	1.23+5	8.45+5	1.18-6

Table I (continued)

n'ℓ' \ n	1	2	3	4	5	6	7	8	9	Total	Lifetime (sec)
100	0.0	1.55+5	1.04+5	7.05+4	5.07+4	3.85+4	3.07+4	2.56+4	2.19+4	4.97+5	2.01-6
101	4.21+6	6.15+5	2.21+5	1.13+5	6.97+4	4.81+4	3.59+4	2.86+4	2.35+4	5.37+6	1.86-7
102	0.0	1.03+6	3.79+5	1.80+5	1.02+5	6.54+4	4.58+4	3.44+4	2.68+4	1.86+6	5.38-7
103	0.0	0.0	3.57+5	2.19+5	1.33+5	8.56+4	5.89+4	4.28+4	3.20+4	9.28+5	1.08-6
104	0.0	0.0	0.0	1.55+5	1.34+5	9.89+4	7.19+4	5.31+4	3.92+4	5.52+5	1.81-6
105	0.0	0.0	0.0	0.0	8.13+4	9.14+4	7.92+4	6.37+4	4.88+4	3.64+5	2.74-6
106	0.0	0.0	0.0	0.0	0.0	5.27+4	7.26+4	7.17+4	6.09+4	2.58+5	3.88-6
107	0.0	0.0	0.0	0.0	0.0	0.0	4.43+4	7.17+4	7.63+4	1.92+5	5.20-6
108	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.35+4	9.53+4	1.49+5	6.72-6
109	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.19+5	1.19+5	8.42-6
Mean	1.26+5	7.13+4	5.16+4	4.24+4	3.80+4	3.69+4	3.91+4	4.68+4	7.16+4	5.24+5	1.91-6

*The fourth digit indicates the power of 10 by which the entry should be raised.

Table II
Branching Rations in Atomic Hydrogen for Transitions in 'ℓ' → n and n' → n

n'ℓ' \ n	1	2	3	4	5
20 21	0.00 1.00				
Mean	7.50-1				
30 31 32	1.00 8.82-1 1.00	1.00 1.18-1 1.00			
Mean	9.61-1	7.06-1			
40 41 42 43	9.51-1 8.81-1 9.70-1 1.00	6.33-1 1.61-1 7.76-1 1.00	4.16-1 4.20-2 2.54-1 1.00		
Mean	9.65-1	7.50-1	5.50-1		
50 51 52 53 54	9.35-1 8.81-1 9.60-1 9.89-1 1.00-1	5.28-1 1.75-1 7.05-1 9.19-1 1.00	3.28-1 5.09-2 2.44-1 7.30-1 1.00	2.27-1 2.20-2 1.07-1 3.63-1 1.00	
Mean	9.72-1	8.00-1	6.33-1	4.95-1	
60 61 62 63 64 65	9.28-1 8.82-1 9.55-1 9.84-1 9.95-1 1.00	4.81-1 1.81-1 6.72-1 8.80-1 9.64-1 1.00	2.87-1 5.49-1 2.37-1 6.39-1 8.79-1 1.00	1.95-1 2.47-2 1.08-1 3.30-1 7.16-1 1.00	1.44-1 1.38-2 5.82-2 1.76-1 4.46-1 1.00
Mean	9.77-1	8.40-1	6.95-1	5.71-1	4.64-1

Table II (continued)

n'ℓ' \ n	1	2	3	4	5	6	7	8
70	9.24-1	4.56-1	2.64-1	1.75-1	1.28-1	9.91-2		
71	8.82-1	1.35-1	5.71-2	2.60-2	1.48-2	9.61-3		
72	9.52-1	6.54-1	2.32-1	1.07-1	5.93-2	3.67-2		
73	9.81-1	8.57-1	5.95-1	3.10-1	1.73-1	1.02-1		
74	9.92-1	9.43-1	8.19-1	6.18-1	3.93-1	2.41-1		
75	9.98-1	9.81-1	9.38-1	8.55-1	7.17-1	5.11-1		
76	1.00	1.00	1.00	1.00	1.00	1.00		
Mean	9.82-1	8.68-1	7.44-1	6.31-1	5.33-1	4.45-1		
80	9.21-1	4.40-1	2.50-1	1.64-1	1.17-1	9.03-2	7.26-2	
81	8.82-1	1.87-1	5.85-2	2.68-2	1.52-2	9.95-3	7.10-3	
82	9.51-1	6.43-1	2.29-1	1.06-1	5.90-2	3.73-2	2.54-2	
83	9.79-1	8.48-1	5.69-1	2.98-1	1.68-1	1.03-1	6.63-2	
84	9.91-1	9.30-1	7.85-1	5.69-1	3.61-1	2.31-1	1.48-1	
85	9.96-1	9.70-1	9.02-1	7.82-1	6.16-1	4.41-1	3.00-1	
86	9.99-1	9.90-1	9.65-1	9.18-1	8.40-1	7.25-1	5.63-1	
87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
Mean	9.85-1	9.91-1	7.82-1	6.80-1	5.86-1	5.06-1	4.32-1	
90	9.20-1	4.29-1	2.41-1	1.56-1	1.10-1	8.35-2	6.70-2	5.56-2
91	8.82-1	1.89-1	5.94-2	2.73-2	1.55-2	1.01-2	7.21-3	5.49-3
92	9.49-1	6.35-1	2.27-1	1.06-1	5.86-2	3.70-2	2.56-2	1.87-2
93	9.77-1	8.34-1	5.53-1	2.89-1	1.64-1	1.01-1	6.72-2	4.65-2
94	9.89-1	9.21-1	7.62-1	5.41-1	3.42-1	2.20-1	1.47-1	9.94-2
95	9.95-1	9.62-1	8.78-1	7.39-1	5.64-1	4.01-1	2.81-1	1.93-1
96	9.98-1	9.82-1	9.43-1	8.69-1	7.60-1	6.22-1	4.80-1	3.51-1
97	9.99-1	9.94-1	9.79-1	9.50-1	9.03-1	8.33-1	7.36-1	6.06-1
98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Mean	9.87-1	9.08-1	8.14-1	7.20-1	6.34-1	5.55-1	4.86-1	4.22-1

Table II (continued)

n'ℓ' \ n	1	2	3	4	5	6	7	8	9
100	9.19-1	4.21-1	2.34-1	1.50-1	1.05-1	7.89-2	6.24-2	5.17-2	4.40-2
101	8.82-1	1.90-1	6.00-2	2.76-2	1.56-2	1.01-2	7.19-3	5.49-3	4.38-3
102	9.49-1	6.30-1	2.26-1	1.05-1	5.84-2	3.67-2	2.53-2	1.87-2	1.44-2
103	9.76-1	8.28-1	5.43-1	2.84-1	1.61-1	9.97-2	6.65-2	4.72-2	3.45-2
104	9.88-1	9.14-1	7.47-1	5.22-1	3.30-1	2.13-1	1.43-1	1.00-1	7.11-2
105	9.94-1	9.56-1	8.62-1	7.11-1	5.32-1	3.76-1	2.65-1	1.89-1	1.34-1
106	9.97-1	9.77-1	9.27-1	8.37-1	7.12-1	5.67-1	4.33-1	3.25-1	2.36-1
107	9.99-1	9.89-1	9.64-1	9.18-1	8.45-1	7.48-1	6.31-1	5.13-1	3.97-1
108	9.99-1	9.96-1	9.87-1	9.68-1	9.38-1	8.93-1	8.31-1	7.47-1	6.40-1
109	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Mean	9.89-1	9.22-1	8.39-1	7.53-1	6.73-1	5.98-1	5.31-1	4.71-1	4.14-1

Tables III. Branching ratios β_T (nℓ, n'ℓ') and β_T (nℓ, n') for $1 \leq n \leq 5$, $0 \leq \ell \leq n-1$, $n < n' \leq 15$, and $0 \leq \ell' \leq n_s$, where n_s is the smaller of 6 and n'-1.

Table III A
Final 1s State

← β_T(1s, n'ℓ') — β_T(1s, n')

n' \ ℓ'	0	1	2	3	4	5	6	Mean
2	0.00	1.00						7.50-1*
3	1.00	8.82-1	1.00					9.61-1
4	9.51-1	8.81-1	9.70-1	1.000				9.65-1
5	9.35-1	8.81-1	9.60-1	9.89-1	1.000			9.72-1
6	9.28-1	8.82-1	9.55-1	9.84-1	9.95-1	1.00		9.77-1
7	9.24-1	8.82-1	9.52-1	9.81-1	9.92-1	9.98-1	1.0000	9.82-1
8	9.21-1	8.82-1	9.51-1	9.79-1	9.91-1	9.96-1	9.99-1	9.85-1
9	9.20-1	8.82-1	9.49-1	9.77-1	9.89-1	9.95-1	9.98-1	9.87-1
10	9.19-1	8.82-1	9.49-1	9.76-1	9.88-1	9.94-1	9.97-1	9.89-1
11	9.18-1	8.82-1	9.48-1	9.76-1	9.88-1	9.93-1	9.96-1	9.91-1
12	9.17-1	8.82-1	9.48-1	9.75-1	9.87-1	9.93-1	9.96-1	9.92-1
13	9.16-1	8.82-1	9.47-1	9.75-1	9.87-1	9.93-1	9.96-1	9.93-1
14	9.16-1	8.82-1	9.47-1	9.75-1	9.87-1	9.92-1	9.96-1	9.94-1
15	9.16-1	8.82-1	9.47-1	9.74-1	9.86-1	9.92-1	9.95-1	9.94-1

*The fourth digit indicates the power of 10 by which the entry should be raised.

Table III B
Final 2s State

$\longleftarrow \beta_T(2s,\ n'\ell') \longrightarrow \beta_T(2s,\ n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
3	0.00	1.18-1	0.00					3.94-2
4	4.92-2	1.19-1	3.01-2	0.00				3.48-2
5	6.48-2	1.19-1	4.03-2	1.09-2	0.00			2.79-2
6	7.20-2	1.18-1	4.51-2	1.63-2	4.87-3	0.00		2.25-2
7	7.60-2	1.18-1	4.78-2	1.94-2	7.70-3	2.49-3	0.00	1.84-2
8	7.85-2	1.18-1	4.95-2	2.13-2	9.49-3	4.08-3	1.40-3	1.53-2
9	8.03-2	1.18-1	5.06-2	2.26-2	1.07-2	5.16-3	2.36-3	1.29-2
10	8.15-2	1.18-1	5.14-2	2.35-2	1.16-2	5.94-3	3.04-3	1.10-2
11	8.24-2	1.18-1	5.20-2	2.42-2	1.22-2	6.50-3	3.55-3	9.46-3
12	8.31-2	1.18-1	5.24-2	2.47-2	1.27-2	6.94-3	3.93-3	8.24-3
13	8.37-2	1.18-1	5.28-2	2.51-2	1.31-2	7.27-3	4.23-3	7.23-3
14	8.41-2	1.18-1	5.30-2	2.54-2	1.34-2	7.54-3	4.46-3	6.40-3
15	8.45-2	1.18-1	5.32-2	2.57-2	1.36-2	7.75-3	4.66-3	5.70-3

Table III C
Final 2p State

$\longleftarrow \beta_T(2p, n'\ell') \longrightarrow$ $\beta_T(2p, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
3	1.00	0.00	1.00					6.67-1
4	5.84-1	4.20-2	7.46-1	1.00				7.15-1
5	4.64-1	5.61-2	6.65-1	9.08-1	1.00			7.72-1
6	4.09-1	6.29-2	6.27-1	8.63-1	9.59-1	1.00		8.17-1
7	3.80-1	6.68-2	6.06-1	8.38-1	9.35-1	9.79-1	1.00	8.50-1
8	3.61-1	6.93-2	5.93-1	8.22-1	9.20-1	9.66-1	9.88-1	8.76-1
9	3.49-1	7.10-2	5.85-1	8.12-1	9.10-1	9.56-1	9.80-1	8.95-1
10	3.40-1	7.23-2	5.79-1	8.04-1	9.03-1	9.50-1	9.74-1	9.11-1
11	3.33-1	7.33-2	5.74-1	7.99-1	8.98-1	9.45-1	9.70-1	9.23-1
12	3.28-1	7.40-2	5.71-1	7.94-1	8.94-1	9.42-1	9.67-1	9.33-1
13	3.24-1	7.46-2	5.68-1	7.91-1	8.91-1	9.39-1	9.64-1	9.41-1
14	3.21-1	7.51-2	5.66-1	7.89-1	8.88-1	9.37-1	9.62-1	9.48-1
15	3.18-1	7.55-2	5.65-1	7.87-1	8.86-1	9.35-1	9.61-1	9.53-1

Table III D
Final 3s State

$\longleftarrow \beta_T(3s, n'\ell') \longrightarrow \beta_T(3s, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
4	0.00	3.77-2	0.00	0.00				7.07-3
5	8.57-3	3.90-2	3.90-3	0.00	0.00			5.80-3
6	1.28-2	3.91-2	5.96-3	6.78-4	0.00	0.00		4.57-3
7	1.52-2	3.91-2	7.15 [illegible]	1.22-3	1.62-4	0.00	0.00	3.64-3
8	1.67-2	3.90-2	7.90-3	1.62-3	3.21-4	4.83-5	0.00	2.94-3
9	1.78-3	3.90-2	8.41-3	1.90-3	4.52-4	1.02-4	1.69-5	2.41-3
10	1.85-2	3.89-2	8.77-3	2.12-3	5.56-4	1.49-4	3.71-5	2.01-3
11	1.90-2	3.89-2	9.04-3	2.28-3	6.38-4	1.90-4	5.63-5	1.70-3
12	1.94-2	3.89-2	9.24-3	2.40-3	7.04-4	2.23-4	7.34-5	1.45-3
13	1.97-2	3.88-2	9.39-3	2.50-3	7.57-4	2.51-4	8.81-5	1.26-3
14	2.00-2	3.88-2	9.52-3	2.58-3	8.00-4	2.74-4	1.01-4	1.10-3
15	2.02-2	3.88-2	9.62-3	2.65-3	8.35-4	2.94-4	1.11-4	9.63-4

Table III E
Final 3p State

$\longleftarrow \beta_T(3p, n'\ell') \longrightarrow \beta_T(3p, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
4	4.16-1	0.00	2.54-1	0.00				1.05-1
5	3.19-1	8.43-3	2.36-1	9.23-2	0.00			8.69-2
6	2.73-1	1.26-2	2.25-1	1.20-1	4.12-2	0.00		7.34-2
7	2.47-1	1.50-2	2.17-1	1.31-1	6.07-2	2.10-2	0.00	6.28-2
8	2.32-1	1.65-2	2.13-1	1.37-1	7.17-2	3.32-2	1.19-2	5.42-2
9	2.21-1	1.75-2	2.10-1	1.41-1	7.86-2	4.09-2	1.95-2	4.72-2
10	2.14-1	1.83-2	2.07-1	1.43-1	8.31-2	4.62-2	2.47-2	4.15-2
11	2.08-1	1.88-2	2.06-1	1.45-1	8.64-2	4.99-2	2.85-2	3.66-2
12	2.04-1	1.92-2	2.04-1	1.46-1	8.87-2	5.27-2	3.13-2	3.26-2
13	2.01-1	1.95-2	2.03-1	1.47-1	9.05-2	5.48-2	3.34-2	2.91-2
14	1.98-1	1.98-2	2.02-1	1.47-1	9.19-2	5.65-2	3.51-2	2.62-2
15	1.96-1	2.00-2	2.02-1	1.48-1	9.30-2	5.78-2	3.64-2	2.37-2

Table III F
Final 3d State

n' \ ℓ'	$\beta_T(3d, n'\ell')$ 0	1	2	3	4	5	6	$\beta_T(3d, n')$ Mean
4	0.00	4.28-3	0.00	1.00				4.38-1
5	9.72-4	3.56-3	3.96-3	6.37-1	1.00			5.40-1
6	1.33-3	3.22-3	6.15-3	5.18-1	8.38-1	1.000		6.17-1
7	1.50-3	3.04-3	7.57-3	4.62-1	7.58-1	9.17-1	1.00	6.78-1
8	1.59-3	2.94-3	8.59-3	4.30-1	7.12-1	8.69-1	9.53-1	7.25-1
9	1.65-3	2.88-3	9.35-3	4.11-1	6.83-1	8.37-1	9.23-1	7.64-1
10	1.68-3	2.84-3	9.96-3	3.97-1	6.63-1	8.16-1	9.02-1	7.95-1
11	1.71-3	2.81-3	1.04-2	3.88-1	6.49-1	8.00-1	8.87-1	8.20-1
12	1.73-3	2.79-3	1.08-2	3.81-1	6.38-1	7.89-1	8.76-1	8.41-1
13	1.75-3	2.77-3	1.12-2	3.75-1	6.30-1	7.80-1	8.67-1	8.59-1
14	1.76-3	2.76-3	1.15-2	3.71-1	6.24-1	7.73-1	8.60-1	8.74-1
15	1.[illegible]-3	2.75-3	1.17-2	3.68-1	6.19-1	7.68-1	8.55-1	8.87-1

Table III G
Final 4s State

$\longleftarrow \beta_T(4s, n'\ell') \longrightarrow \beta_T(4s, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
5	0.00	1.75-2	0.00	0.00	0.00			2.10-3
6	2.52-3	1.82-2	9.39-4	0.00	0.00	0.00		1.72-3
7	4.02-3	1.83-2	1.54-3	9.19-5	0.00	0.00	0.00	1.37-3
8	4.97-3	1.82-2	1.92-3	1.85-4	1.33-5	0.00	0.00	1.11-3
9	5.61-3	1.82-2	2.19-3	2.63-4	3.02-5	2.53-6	0.00	9.05-4
10	6.06-3	1.82-2	2.38-3	3.25-4	4.65-5	6.22-6	5.90-7	7.52-4
11	6.39-3	1.81-2	2.51-3	3.75-4	6.10-5	1.02-5	1.54-6	6.34-4
12	6.65-3	1.81-2	2.62-3	4.14-4	7.35-5	1.39-5	2.62-6	5.41-4
13	6.85-3	1.81-2	2.70-3	4.46-4	8.40-5	1.73-5	3.71-6	4.66-4
14	7.01-3	1.81-2	2.77-3	4.73-4	9.30-5	2.04-5	4.76-6	4.06-4
15	7.14-3	1.81-2	2.82-3	4.94-4	1.01-4	2.30-5	5.71-6	3.56-4

Table III H
Final 4p State

n' \ ℓ'	0	1	2	3	4	5	6	Mean
	$\longleftarrow$ $\beta_T(4p, n'\ell')$						$\longrightarrow$	$\beta_T(4p, n')$
5	2.27-1	0.00	1.04-1	0.00	0.00			2.98-2
6	1.92-1	2.66-3	1.03-1	1.80-2	0.00	0.00		2.33-2
7	1.70-1	4.23-3	1.00-1	2.70-2	4.30-3	0.00	0.00	1.86-2
8	1.57-1	5.22-3	9.84-2	3.20-2	7.72-3	1.28-3	0.00	1.52-2
9	1.49-1	5.88-3	9.70-2	3.52-2	1.02-2	2.54-3	4.48-4	1.26-2
10	1.43-1	6.35-3	9.60-2	3.72-2	1.20-2	3.59-3	9.49-4	1.06-2
11	1.38-1	6.69-3	9.52-2	3.87-2	1.33-2	4.43-3	1.40-3	9.09-3
12	1.35-1	6.96-3	9.46-2	3.97-2	1.44-2	5.10-3	1.79-3	7.84-3
13	1.33-1	7.16-3	9.41-2	4.05-2	1.52-2	5.64-3	2.12-3	6.82-3
14	1.30-1	7.33-3	9.37-2	4.11-2	1.58-2	6.08-3	2.39-3	5.99-3
15	1.29-1	7.46-3	9.34-2	4.16-2	1.63-2	6.44-3	2.62-3	5.29-3

Table III I
Final 4d State

$\longleftarrow \beta_T(4d, n'\ell') \longrightarrow \beta_T(4d, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
5	0.00	4.48-3	0.00	3.63-1	0.00			1.02-1
6	6.43-4	3.84-3	1.93-3	3.09-1	1.62-1	0.00		1.01-1
7	9.49-4	3.49-3	3.13-3	2.78-1	2.00-1	8.27-2	0.00	9.56-2
8	1.11-3	3.28-3	3.94-3	2.60-1	2.13-1	1.19-1	4.66-2	8.88-2
9	1.21-3	3.15-3	4.53-3	2.48-1	2.18-1	1.38-1	7.26-2	8.17-2
10	1.28-3	3.06-3	4.97-3	2.40-1	2.21-1	1.50-1	8.87-2	7.50-2
11	1.32-3	3.00-3	5.32-3	2.34-1	2.22-1	1.57-1	9.94-2	6.87-2
12	1.35-3	2.96-3	5.60-3	2.29-1	2.22-1	1.62-1	1.07-1	6.29-2
13	1.38-3	2.92-3	5.84-3	2.26-1	2.23-1	1.65-1	1.12-1	5.77-2
14	1.40-3	2.90-3	6.03-3	2.23-1	2.23-1	1.68-1	1.17-1	5.30-2
15	1.41-3	1.41-3	2.88-3	2.21-1	2.23-1	1.70-1	1.20-1	4.88-2

Table III J
Final 4f State

$\longleftarrow \beta_T(4f, n'\ell') \longrightarrow \beta_T(4f, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
5	0.00	0.00	3.52-3	0.00	1.00			3.61-1
6	0.00	1.38-5	2.56-3	3.34-3	5.54-1	1.00		4.45-1
7	1.36-6	2.04-5	2.12-3	4.90-3	4.13-1	7.72-1	1.00	5.15-1
8	2.72-6	2.43-5	1.90-3	5.83-3	3.48-1	6.61-1	8.72-1	5.75-1
9	3.84-6	2.67-5	1.77-3	6.47-3	3.12-1	5.98-1	7.96-1	6.25-1
10	4.75-6	2.85-5	1.69-3	6.95-3	2.89-1	5.58-1	7.48-1	6.67-1
11	5.48-6	2.98-5	1.64-3	7.33-3	2.74-1	5.30-1	7.14-1	7.03-1
12	6.07-6	3.08-5	1.60-3	7.64-3	2.63-1	5.11-1	6.91-1	7.34-1
13	6.56-6	3.16-5	1.57-3	7.90-3	2.55-1	4.96-1	6.73-1	7.61-1
14	6.97-6	3.23-5	1.55-3	8.12-3	2.49-1	4.85-1	6.59-1	7.84-1
15	7.31-6	3.29-5	1.54-3	8.31-3	2.44-1	4.76-1	6.48-1	8.03-1

Table III K
Final 5s State

	$\beta_T(5s, n'\ell')$							$\beta_T(5s, n')$
n' \ ℓ'	0	1	2	3	4	5	6	Mean
6	0.00	9.92-3	0.00	0.00	0.00	0.00		8.26-4
7	9.82-4	1.03-2	3.17-4	0.00	0.00	0.00	0.00	6.81-4
8	1.63-3	1.03-2	5.41-4	1.95-5	0.00	0.00	0.00	5.52-4
9	2.07-3	1.03-2	6.96-4	4.22-5	1.85-6	0.00	0.00	4.52-4
10	2.38-3	1.02-2	8.08-4	6.30-5	4.62-6	2.40-7	0.00	3.76-4
11	2.61-3	1.02-2	8.90-4	8.08-5	7.60-6	6.59-7	3.96-8	3.16-4
12	2.78-3	1.02-2	9.53-4	9.57-5	1.05-5	1.16-6	1.16-7	2.70-4
13	2.91-3	1.01-2	1.00-3	1.08-4	1.31-5	1.69-6	2.16-7	2.32-4
14	3.02-3	1.01-2	1.04-3	1.19-4	1.54-5	2.21-6	3.28-7	2.02-4
15	3.11-3	1.01-2	1.07-3	1.27-4	1.75-5	2.69-6	4.42-7	1.77-4

Table III L
Final 5p State

$\longleftarrow \beta_T(5p, n'\ell') \longrightarrow \beta_T(5p, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
6	1.44-1	0.00	5.35-2	0.00	0.00	0.00		1.14-2
7	1.27-1	1.08-3	5.44-2	5.24-3	0.00	0.00	0.00	8.96-3
8	1.15-1	1.79-3	5.38-2	8.52-3	7.58-4	0.00	0.00	7.12-3
9	1.07-1	2.26-3	5.32-2	1.06-2	1.53-3	1.44-4	0.00	5.80-3
10	1.02-1	2.59-3	5.26-2	1.20-2	2.17-3	3.29-4	3.37-5	4.81-3
11	9.84-2	2.83-3	5.22-2	1.30-2	2.69-3	5.10-4	8.35-5	4.05-3
12	9.55-2	3.02-3	5.19-2	1.38-2	3.10-3	6.72-4	1.37-4	3.46-3
13	9.34-2	3.16-3	5.16-2	1.43-2	3.43-3	8.12-4	1.89-4	2.98-3
14	9.16-2	3.27-3	5.14-2	1.47-2	3.70-3	9.32-4	2.37-4	2.60-3
15	9.02-2	3.36-3	5.12-2	1.51-2	3.92-3	1.03-3	2.80-4	2.28-3

Table III M
Final 5d State

$\longleftarrow \beta_T(5d, n'\ell') \longrightarrow \beta_T(5d, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
6	0.00	3.92-3	0.00	1.74-1	0.00	0.00		3.41-2
7	3.88-4	3.42-3	9.47-4	1.63-1	4.15-2	0.00	0.00	3.13-2
8	5.99-4	3.12-3	1.58-3	1.54-1	6.06-2	1.24-2	0.00	2.78.2
9	7.24-4	2.93-3	2.03-3	1.48-1	7.05-2	2.19-2	4.33-3	2.45-2
10	8.03-4	2.80-3	2.35-3	1.43-1	7.63-2	2.86-2	8.55-3	2.16-2
11	8.58-4	2.72-3	2.61-3	1.39-1	7.99-2	3.35-2	1.20-2	1.91-2
12	8.97-4	2.65-3	2.80-3	1.37-1	8.24-2	3.70-2	1.48-2	1.69-2
13	9.26-4	2.61-3	2.97-3	1.35-1	8.41-2	3.97-2	1.70-2	1.51-2
14	9.49-4	2.57-3	3.10-3	1.33-1	8.54-2	4.17-2	1.87-2	1.35-2
15	9.67-4	2.54-3	3.21-3	1.32-1	8.63-2	4.33-2	2.02-2	1.21-2

Table III N
Final 5f State

ℓ' / n'	$\beta_T(5f, n'\ell')$							$\beta_T(5d, n')$
	0	1	2	3	4	5	6	Mean
6	0.00	0.00	4.66-3	0.00	4.46-1	0.00		1.12-1
7	0.00	1.53-5	3.59-3	2.33-3	3.48-1	2.28-1	0.00	1.16-1
8	1.11-6	2.35-5	3.04-3	3.62-3	2.96-1	2.68-1	1.28-1	1.14-1
9	2.32-6	2.83-5	2.73-3	4.45-3	2.66-1	2.75-1	1.79-1	1.11-1
10	3.40-6	3.15-5	2.54-3	5.02-3	2.47-1	2.74-1	2.03-1	1.05-1
11	4.30-6	3.37-5	2.41-3	5.45-3	2.33-1	2.72-1	2.16-1	9.99-2
12	5.05-6	3.54-5	2.32-3	5.79-3	2 24-1	2.69-1	2.23-1	9.41-2
13	5.67-6	3.67-5	2.25-3	6.07-3	2.17-1	2.66-1	2.28-1	8.84-2
14	6.19-6	3.78-5	2.20-3	6.29-3	2.12-1	2.64-1	2.31-1	8.30-2
15	6.64-6	3.86-5	2.17-3	6.49-3	2.08-1	2.62-1	2.33-1	7.78-2

Table III O
Final 5g State

← $\beta_T(5g, n'\ell')$ → $\beta_T(5g, n')$

n' \ ℓ'	0	1	2	3	4	5	6	Mean
6	0.00	0.00	0.00	2.73-3	0.00	1.00		3.06-1
7	0.00	0.00	1.29-5	1.76-3	2.78-3	4.89-1	1.00	3.76-1
8	0.00	3.56-8	1.81-5	1.35-3	3.84-3	3.36-1	7.12-1	4.37-1
9	1.98-9	7.39-8	2.07-5	1.14-3	4.37-3	2.67-1	5.76-1	4.92-1
10	5.08-9	1.08-7	2.22-5	1.03-3	4.72-3	2.29-1	5.00-1	5.41-1
11	8.58-9	1.37-7	2.32-5	9.56-4	4.96-3	2.06-1	4.52-1	5.83-1
12	1.21-8	1.61-7	2.40-5	9.08-4	5.15-3	1.90-1	4.20-1	6.21-1
13	1.54-8	1.82-7	2.45-5	8.75-4	5.31-3	1.80-1	3.97-1	6.54-1
14	1.85-8	1.99-7	2.50-5	8.52-4	5.45-3	1.71-1	3.80-1	6.83-1
15	2.16-8	2.14-7	2.54-5	8.35-4	5.56-3	1.65-1	3.67-1	7.09-1